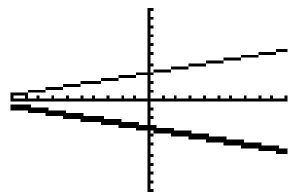
1.3 Reflecting Graphs of Functions

Invariant points: Points on a graph which do not move after a transformation

Using a graphing calculator, graph $y = f(x)$ and $y = -f(x)$
$f(x) = (1/4)x + 3$
$-f(x) = (-1/4)x - 3$



Reflection across the x-axis.

Which axis is f(x) reflected in?

Let's look at a table of values for this graph

X	Y1	Y2
-3	2.25	-2.25
-2	2.5	-2.5
-1	2.75	-2.75
0	3	-3
1	3.25	-3.25
2	3.5	-3.5
3	3.75	-3.75

X= -3

Notice that every y value in f(x) is replaced with -y in -f(x)

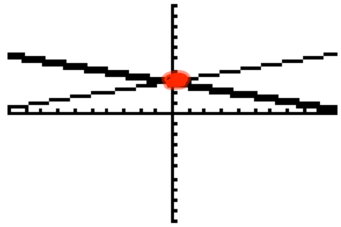
Reflections in the x-axis:
1. Each y -value of f(x) is multiplied by -1
2. The point (x, y) in f(x) becomes (x, -y) for -f(x)
3. All x - intercepts stay the same. **They are invariant points**
4. To find the equation for -f(x) multiply f(x) by -1

$f(x)$		$-f(x)$
$(2, 3)$	$\longrightarrow$	$(2, -3)$
$(4, -6)$	$\longrightarrow$	$(4, 6)$
$(-3, 7)$	$\longrightarrow$	$(-3, -7)$

Using a graphing calculator, graph $y = f(x)$ and $y = f(-x)$

$f(x) = (1/4)x + 3$

$f(-x) = (1/4)(-x) + 3$



Reflection in y-axis.

Which axis is f(x) reflected in?

Let's look at a table of values for this graph

X	Y1	Y2
-3	2.25	3.75
-2	2.5	3.5
-1	2.75	3.25
0	3	3
1	3.25	2.75
2	3.5	2.5
3	3.75	2.25

X= -3

Notice that 3 and -3 have the same y - value. So, the y-values remain the same and the x values are multiplied by -1.

Reflections in y - axis

1. The point (x, y) in f(x) becomes (-x, y) for f(-x)
2. The y - intercepts remain the same. **They are invariant points.**
3. To find the equation for f(-x) substitute -x into f(x) for x.

$f(x)$		$f(-x)$
$(-7, 6)$	$\longrightarrow$	$(7, 6)$
$(-2, -4)$	$\longrightarrow$	$(2, -4)$

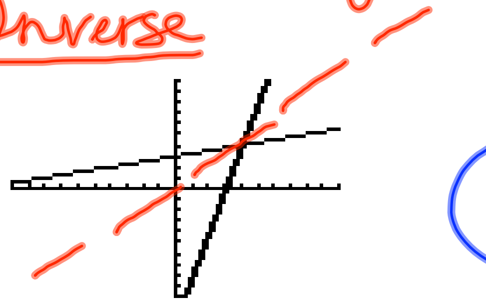
Using a graphing calculator, graph y = f(x) and x = f(y).**Note:** To graph x = f(y) you will have to solve for y.

$y = f^{-1}(x)$

f(x) = (1/4)x + 3 x = f(y)

Inverse

y=x



Solve for y

$x = \frac{1}{4}(y) + 3$

$x - 3 = \frac{1}{4}(y)$

$4(x-3) = y$

This is called the **graph of the inverse.** In such a situation, the coordinates of x and y in y = f(x) are interchanged to get the function x = f(y). This results in a reflection in the line y = x. The graph of the inverse is often written as $y = f^{-1}(x)$ instead of x = f(y).

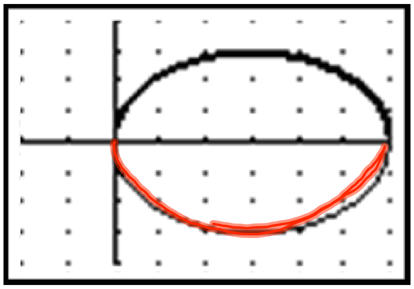
Relfection in the line y = x

1. The point (x, y) in f(x) becomes (y, x) for f(y)
2. To find the equation of the inverse, interchange x and y and solve for y.

f(x) x=f(y)

(4,5) → (5,4)

Each graph is the reflection of the other in the x-axis. Write the equation of the graph illustrated by the wide, dark curve if the equation of normal curve is

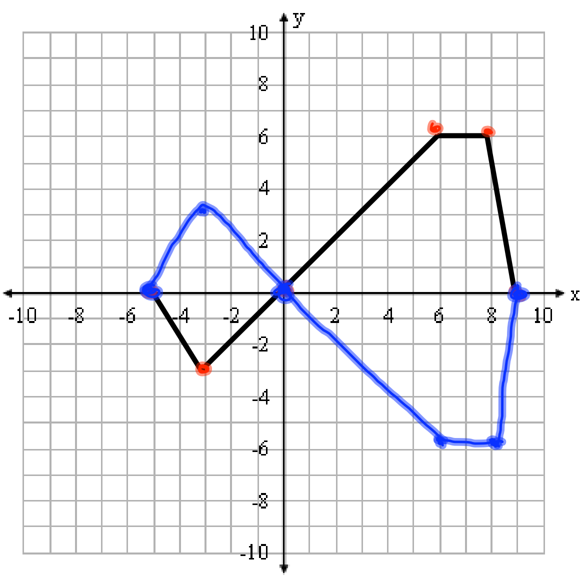


$y = -\sqrt{9-(x-3)^2}$

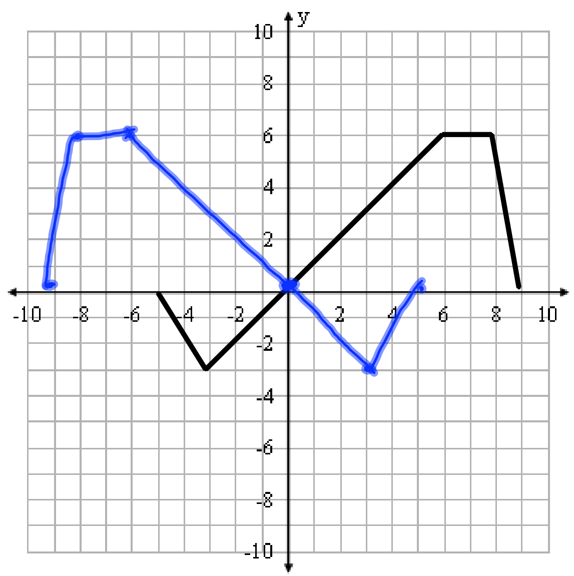
$y = \sqrt{9-(x-3)^2}$

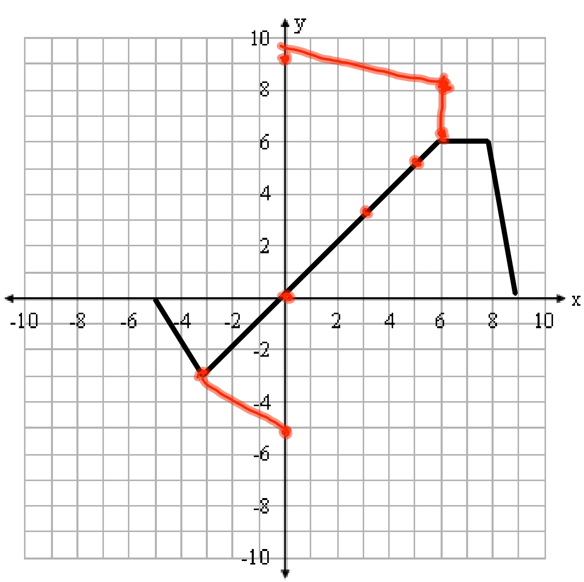
For the following function f(x) graph

y = -f(x)



y = f(-x)





x = f(y)

For each of the following state the equation for x = f(y)

$$y = \frac{1}{8}x^3 - 1$$

$$x = \frac{1}{8}y^3 - 1$$

$$x + 1 = \frac{1}{8}y^3$$

$$8(x+1) = y^3$$

$$\sqrt[3]{8(x+1)} = y$$

$$f(x) = \sqrt{3 - x}$$

$$y = \sqrt{3 - x}$$

$$x = \sqrt{3 - y}$$

$$x^2 = 3 - y$$

$$x^2 - 3 = -1y$$

$$-x^2 + 3 = y$$

or

$$3 - x^2 = y$$

$$y = \frac{1}{x^2 - 1}$$

$$\frac{x}{1} = \frac{1}{y^2 - 1}$$

$$1 = x(y^2 - 1)$$

$$\frac{1}{x} = y^2 - 1$$

$$\frac{1}{x} + 1 = y^2$$

$$\pm\sqrt{\frac{1}{x} + 1} = y$$

Pg. 31
2, 4, 5-8
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