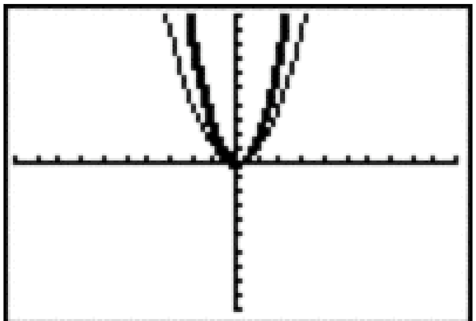
1.4 Stretching Graphs of Functions

Graph $y = x^2$ and $y = 2x^2$ on the same axis and look at the table of values of these two functions

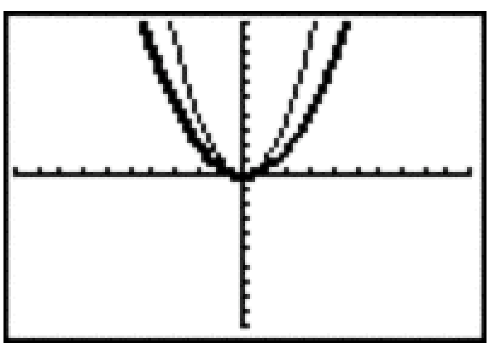


X	Y1	Y2
0	0	0
1	1	2
2	4	8
3	9	18
4	16	32
5	25	50
6	36	72

X=0

Notice how each of the y coordinates for y2 is double those of y1. The point (x, y) has become the point (x, 2y). We say that the graph of $y = x^2$ is **expanded vertically** by a factor of 2.

Graph $y = x^2$ and $y = \frac{1}{2}x^2$ on the same axis and look at the table of values of these two functions.



X	Y1	Y2
0	0	0
1	1	.5
2	4	2
3	9	4.5
4	16	8
5	25	12.5
6	36	18

X=0

Notice how each of the y coordinates of y2 is one-half those of y1. The point (x, y) has become (x, 0.5y). We say that the graph of $y = x^2$ is **compressed vertically** by a factor of one-half.

What would happen to the graph of the function if instead of using positive values in front of the function we used negative values?

For example $y = -2x^2$.

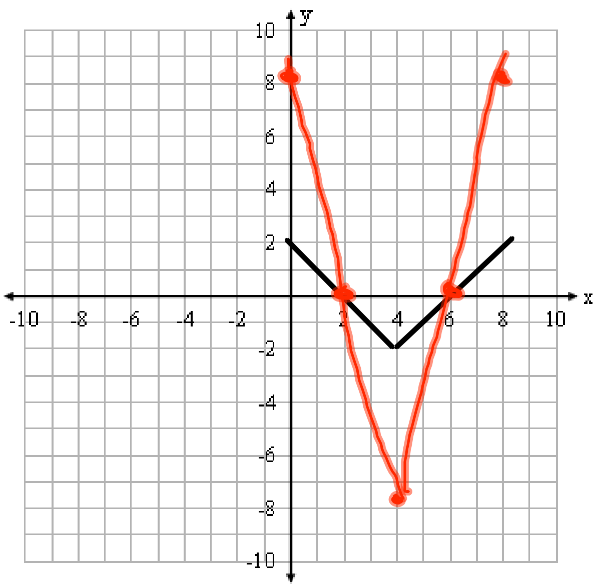
$y = -f(x)$

- vertically expanded by a factor of 2
- reflected in the x-axis

Vertical Stretching:

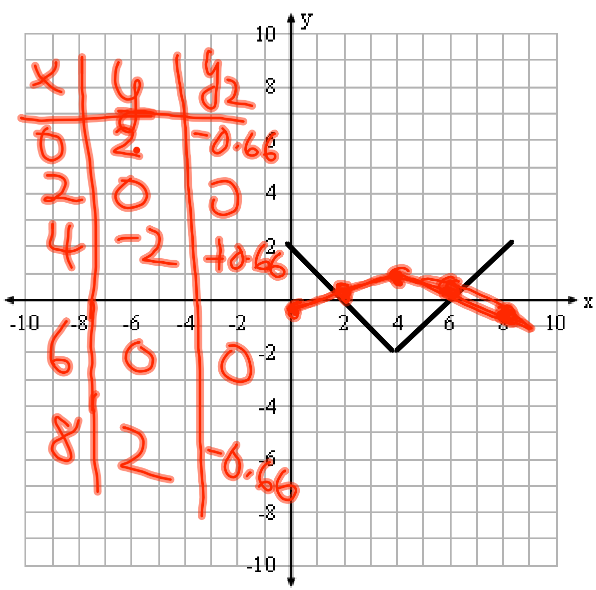
1. In general, for any function $y = f(x)$ the graph of the function $y = af(x)$ is obtained by multiplying the y – value of every point in $y = f(x)$ by "a".
2. For (x, y) in $f(x)$ the points become (x, ay) for $af(x)$
3. If $a>1$ the graph of $y = f(x)$ is expanded vertically by a factor of "a"
4. If $0<a<1$ the graph of $y = f(x)$ is compressed vertically by a factor of "a".
5. If $a<0$ the graph is reflected in the x – axis.
6. All x – intercepts remain the same. **They are invariant points**.

Describe how each of the following functions relates to y = f(x) and then draw the graph of the compression of expansion on the grid.



y = 4 f(x)

- expanded vertically by a factor of 4.

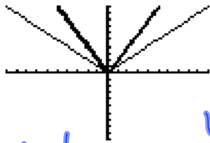


x	y	y_2
0	2	-0.66
2	0	0
4	-2	+0.66
6	0	0
8	2	-0.66

y = (-1/3) f(x)

- reflected in x-axis
- compressed vertically by a factor of $\frac{1}{3}$.

Graph y = |x| and y = |2x| on the same axis and look at a table of values.
Also notice the different position of our coefficient.



$y = |2x|$

$2 \rightarrow \frac{1}{2}$

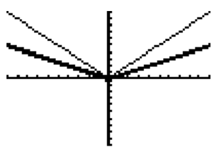
You must take the reciprocal of this value <u>before</u> you decide if it is an expansion or compression.

Notice how of y = |2x| is compressed horizontally. This is because every point (x, y) on y = |x| is transformed to (x/2, y) on y = |2x|. This is a horizontal compression by a factor of 1/2.

Graph y = |x| and y = |1/2 x| on the same axis and look at a table of values.

$\frac{1}{2} \rightarrow 2$

Expand horizontally by a factor of 2.



Notice how the graph of y = |1/2 x| is expanded horizontally. This is because every point (x, y) on y = |x| is transformed to (2x, y) on y = |1/2 x|. This is a horizontal expansion by a factor of 2.

What would happen to the graph of the function if you used negative values next to x instead of positive values?

For example: $y = (-3x)^3$

$y = f(-x)$

- reflection in y - axis

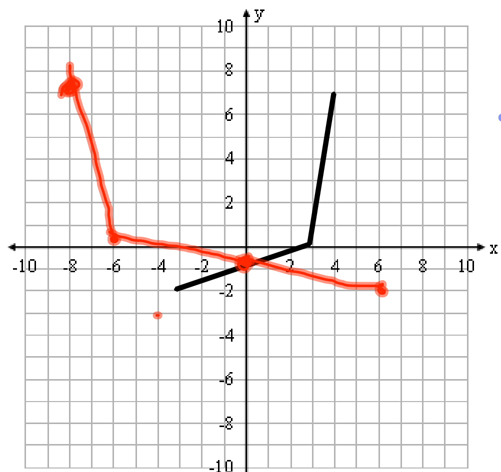
$3 \rightarrow \frac{1}{3}$

- compressed horizontally by a factor of $\frac{1}{3}$.

Horizontal Stretching:

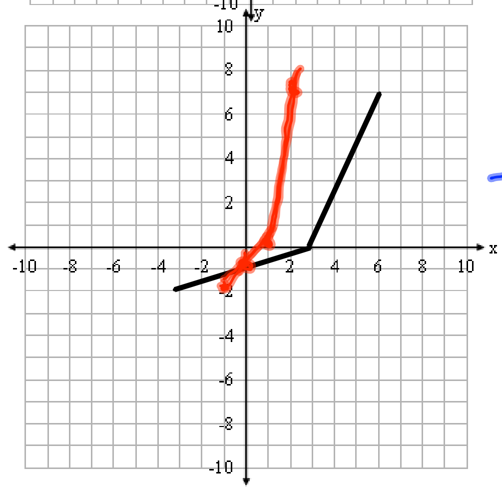
1. The point (x, y) on the graph of the function $y = f(x)$ becomes the point $(x/k, y)$ on the graph of the function $y = f(kx)$.
2. If $k>1$ the graph of $y = f(x)$ is compressed horizontally by a factor of $1/k$.
3. If $0<k<1$ the graph of $y = f(x)$ is expanded horizontally by a factor of $1/k$.
4. If $k<0$ then the graph is reflected in the y - axis.
5. The y - intercepts remain the same. **They are invariant points.**

Describe how each of the following functions relates to y = f(x) and then draw the graph of the compression of expansion on the grid.



y = f(-1/2x)

- reflected in y-axis
- horizontally expanded by a factor of 2.



y = f(3x)

- horizontally compressed by a factor of $\frac{1}{3}$

Pg. 41 2, 3, 5, 8, 10, 18