2.12 Logarithmic Equations and Identities

$x + \log 0.0826 = \log 8.76$

$x = \log 8.76 - \log 0.0826$

$x = 2.03$

$2 \log x = \log 16$

$\log x^2 = \log 16$

Remove the log

$x^2 = 16$

$\sqrt{x^2} = \pm\sqrt{16}$

$x = 4$ $\quad \cancel{x = -4}$

We cannot evaluate the log of any negative value.

$\log(2x) + \log(x+5) = 2$

$\log_{10}(2x)(x+5) = 2$

Convert this to exponential form.

$2x(x+5) = 10^2$

$2x^2 + 10x = 100$

$2x^2 + 10x - 100 = 0$

$2(x^2 + 5x - 50) = 0$

$\boxed{5 \atop 10}x - 50$

$+5$

$\frac{10}{1}$ $\frac{-5}{1}$

$(x+10)(x-5) = 0$

$x+10=0$ $\quad$ $x-5=0$

$\cancel{x=-10}$ $\quad$ $x=5$

$$\frac{1}{2}\log_6 9 = \log_6 x - \log_6 27^{\frac{1}{3}}$$

$\log_6 9^{\frac{1}{2}} = \log_6\left(\frac{x}{27^{1/3}}\right)$

$\log_6 3 = \log_6\left(\frac{x}{3}\right)$

$3 = \frac{x}{3}$

$x = 9$

$$\log_3(x-2)+\log_3(x-3)=2$$

$$\log_3 (x-2)(x-3)=2$$

$$(x-2)(x-3)=3^2$$

$$x^2-3x-2x+6=9$$

$$x^2-5x-3=0$$

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$ax^2+bx+c$$

$$x=\frac{5\pm\sqrt{(5)^2-4(1)(3)}}{2(1)}$$

$$x=\frac{5\pm\sqrt{25+12}}{2}$$

$$x=\frac{5\pm\sqrt{37}}{2}$$

$$x=\frac{5+\sqrt{37}}{2}$$

$$\cancel{x=\frac{5-\sqrt{37}}{2}}$$

$$\log_8(2-x)+\log_8(4-x)=1$$

$$\log_8 (2-x)(4-x)=1$$

$$(2-x)(4-x)=8^1$$

$$8-4x-2x+x^2=8$$

$$x^2-6x=0$$

$$x(x-6)=0$$

$$x=0 \qquad x-6=0$$

$$x=6$$

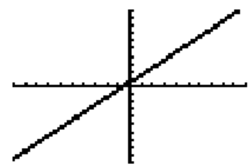
Consider the following identity $\log_a 5 + x = \log_a(5a^x)$

a) Verify the identity numerically when a = 10 and when x = 2

$$\log_{10} 5 + 2 = \log_{10}(5 \cdot 10^2)$$
$$2.69897 = 2.69897$$

b) Verify the identity graphically when a = 10.

$$y_1 = \log_{10} 5 + x$$
$$y_2 = \log_{10}(5 \cdot 10^x)$$



c) Prove the identity for any positive base "a" and any value "x".

$$\log_a 5 + x = \log_a(5 \cdot a^x)$$
$$= \log_a 5 + x \log_a a$$
$$= \log_a 5 + x \quad ✓$$

Pg. 152 2-8 odds