

## Day 2: More Solving Equations

Continued from last day:

$$2 \sin x = 3 + 2 \csc x \text{ for } 0 \leq x \leq 2\pi$$

$$2 \overset{\sin x}{\cancel{\sin x}} = 3 + 2 \cdot \left( \frac{1}{\cancel{\sin x}} \right) \overset{\sin x}{\cancel{\sin x}}$$

$$2 \sin^2 x = 3 \sin x + 2$$

$$2 \sin^2 x - 3 \sin x - 2 = 0$$

$$(2 \sin x + 1) (\sin x - 2) = 0$$

$$\sin x = -\frac{1}{2}$$

$$\left( \frac{7\pi}{6}, \frac{11\pi}{6} \right)$$

~~$$\sin x = 2$$~~

$$\sec^4 \theta - 1 = 0 \text{ for } 0 \leq \theta \leq 2\pi$$

$$(\sec^2 \theta + 1) (\sec^2 \theta - 1) = 0$$

$$(\sec^2 \theta + 1)$$

$$\sec^2 \theta + 1 = 0$$

~~$$\sec^2 \theta = -1$$~~

$$(\sec \theta + 1) (\sec \theta - 1) = 0$$

$$\sec \theta = -1$$

$$\cos \theta = -1$$

$$\pi$$

$$\sec \theta = 1$$

$$\cos \theta = 1$$

$$0, 2\pi$$

$$2 \sin x \cos x + 2 \sin x = \cos x + 1 \text{ for } 0 \leq x \leq 2\pi$$

$$2 \sin x (\cos x + 1) = (\cos x + 1)$$

$$2 \sin x (\cos x + 1) - (\cos x + 1) = 0$$

$$(\cos x + 1)(2 \sin x - 1) = 0$$

$$\cos x = -1$$

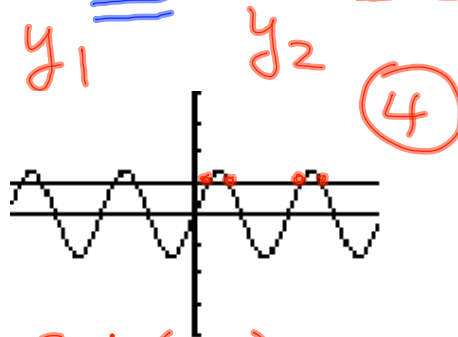
$$\uparrow$$

$$\sin x = \frac{1}{2}$$

$$\frac{\pi}{6}, \frac{5\pi}{6}$$

Fig. 1) Solve the following multiple angle equations in exact form. First graph the equations to determine the number of solutions to each problem.

$$\sqrt{2} \sin(2\theta) = 1 \text{ for } 0 \leq \theta \leq 2\pi$$



$$\sqrt{2} \sin(2\theta) = 1$$

$$\beta = 2\theta \quad 0 \leq \theta \leq 2\pi$$

$$\sqrt{2} \cdot \sin(\beta) = 1$$

$$\sin \beta = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$$

$$0 \leq 2\theta \leq 4\pi$$

$$\beta = \frac{3\pi}{4}, \frac{\pi}{4}$$

$$\frac{\pi}{4} + 2\pi \times 4$$

$$\frac{3\pi}{4} + 2\pi$$

$$\frac{\pi}{4} + 8\pi$$

$$\frac{3\pi}{4} + 8\pi$$

$$\theta = \frac{\pi}{8}, \frac{3\pi}{8}, \frac{9\pi}{8}, \frac{11\pi}{8}$$

$$2 \sin(4\theta) + 1 = 0 \text{ for } 0 \leq \theta \leq 2\pi$$

$$\beta = 4\theta \quad 0 \leq 4\theta \leq 8\pi$$

$$2 \sin \beta + 1 = 0$$

$$2\pi = \frac{12\pi}{6}$$

$$\sin \beta = -\frac{1}{2}$$

$$4\theta = \beta = \frac{7\pi}{6}, \frac{11\pi}{6}, \frac{19\pi}{6}, \frac{23\pi}{6}, \frac{31\pi}{6}, \frac{35\pi}{6}$$

$$\frac{43\pi}{6}, \frac{47\pi}{6}$$

$$\theta = \frac{7\pi}{24}, \frac{11\pi}{24}, \frac{19\pi}{24}, \frac{23\pi}{24}, \frac{31\pi}{24}, \frac{35\pi}{24}, \frac{43\pi}{24}, \frac{47\pi}{24}$$

$$4 \sin^2(2\theta) = 3 \text{ for } 0 \leq \theta \leq \pi$$

$$\beta = 2\theta$$

$$0 \leq 2\theta \leq 2\pi$$

$$4 \sin^2(\beta) = 3 \implies \sin^2(\beta) = \frac{3}{4}$$

$$\sin(\beta) = \pm \frac{\sqrt{3}}{2}$$

$$\sin \beta = \frac{\sqrt{3}}{2}$$

$$\sin \beta = -\frac{\sqrt{3}}{2}$$

$$\beta = \frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}$$

Assignment:  
Handout