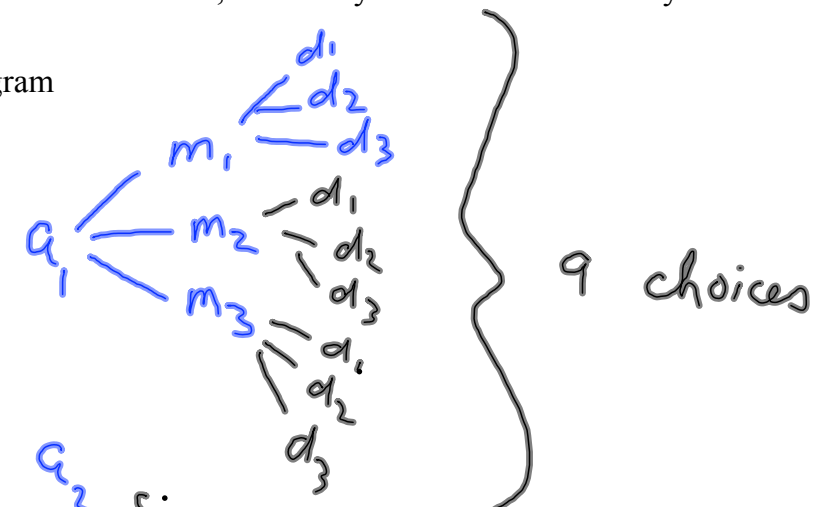
Day 1: Fundamental Counting Principle and Permutations

Fundamental Counting Prinicple: Counting without counting!!!

Eg 1) If Connor were at a restaurant with 2 appetizer choices, 3 main courses and 3 desserts, how many different meals could you order?

Tree Diagram



a_2 Similarly $\Rightarrow$ 9 choices

FCP

TOTAL = 18

$2 \times 3 \times 3 = 18$

appet. main dessert

Definition (FCP): If there are a_1 ways of making a first choice, a_2 ways of making a second a_3 ways of making a third and so on, the total number of ways of making ALL choices is $a_1 \times a_2 \times a_3 \times \ldots$

Eg 2) She has 10 dresses, 2 blouses, 12 pairs shoes and 5 coats. How many different ways can she dress?

$10 \times 2 \times 12 \times 5 = 1200$

Eg 3) How many even 2 digit numbers are there?

$9 \times 5 = 45$

(no tens 0) ↑ ↑ ones

Eg 4) How many 2 digit whole numbers can be formed using the digits 0, 1, 2, 4, 6, 7, 8, 9?

$7 \times 8 = 56$

How many if you cannot repeat a digit in any given whole number?

$7 \times 7 = 49$

no 0 — Can use 0 but not what is in tens spot

Eg 5) At an elementary school track meet, all participants are given ribbons based on their 'place'. If all 8 runners get place ribbons, in how many different ways could the runners receive their ribbons?

$8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1 = 40320$

(8: 1st place; 7: 2nd place; 1: 8th place)

This product can be written as $8!$

"8 factorial"

CALCULATOR

8 MATH PRB 4 ENTER = 40320

$n! = n(n-1)(n-2) \ldots 3 \cdot 2 \cdot 1$

This product is called 8! (aka a permutation)

Permutation - An arrangement of items where order must be considered.

Eg 6) In how many different ways can you arrange the letters BAIT?

eg BAIT
ABIT
TIBA
etc.

$4 \times 3 \times 2 \times 1 = 24$

OR $4! = 24$

* NOTE : n different things can be arranged in a row in n! ways

Eg 7) If letters are not repeated, in how many different ways can you form a 3 letter permutation using the alphabet?

$26 \times 25 \times 24 = 15600$

PERMUTATION KEY ON CALCULATOR

26 nPr 3 = 15600

MATH PRB → 26P3

Can solve such permutation questions with a formula:

$${}_nP_r = \frac{n!}{(n-r)!}$$

$${}_{26}P_3 = \frac{26!}{(26-3)!} = \frac{26!}{23!} = 15600$$

$$= \frac{26 \times 25 \times 24 \times 23!}{23!}$$

Eg 8) How many different 3 letter permutations can be formed with the word keyboard?

$8 \times 7 \times 6$

$= 336$

${}_8P_3 = 336$

Special Case: ${}_nP_n = n!$

eg ${}_5P_5 = 120$

$5! = 5 \times 4 \times 3 \times 2 \times 1 = 120$

Eg 9) In a certain country, license plates are formed by 3 different numbers following by 4 different letters. How many different plates are possible?

$10 \times 9 \times 8 \times 26 \times 25 \times 24 \times 23$

$= 258\,336\,000$

OR $_{10}P_3 \times {}_{26}P_4$

Eg 10) If phone numbers are made up of 3 different digits following by 4 different digits, how many different phone numbers are possible? How many are possible if the digits don't need to be different?

a) $10 \times 9 \times 8 \times 10 \times 9 \times 8 \times 7$

$= 3\,628\,800$

b) $10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10$

$= 10^7 = 10\,000\,000$

Simplify $\dfrac{(n-2)!}{n!} = \dfrac{(n-2)!}{n(n-1)(n-2)!} = \dfrac{1}{n(n-1)}$

Solve the equation ${}_nP_2 = 20$

$nPr = \dfrac{n!}{(n-r)!}$

$\dfrac{n!}{(n-2)!} = 20$

$\dfrac{n(n-1)(n-2)!}{(n-2)!} = 20$

$n(n-1) = 20$

$5 \times 4 = 20 \Rightarrow n = 5$

$n^2 - n - 20 = 0$

$(n-5)(n+4) = 0$

$n = 5$ OR $n = -4$ ← extraneous

Assignment

Pg. 356 1b, 3, 5, 11ac, 13

Pg. 364 4-6, 8, 9, 12, 13, 18odds, 21 odds

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