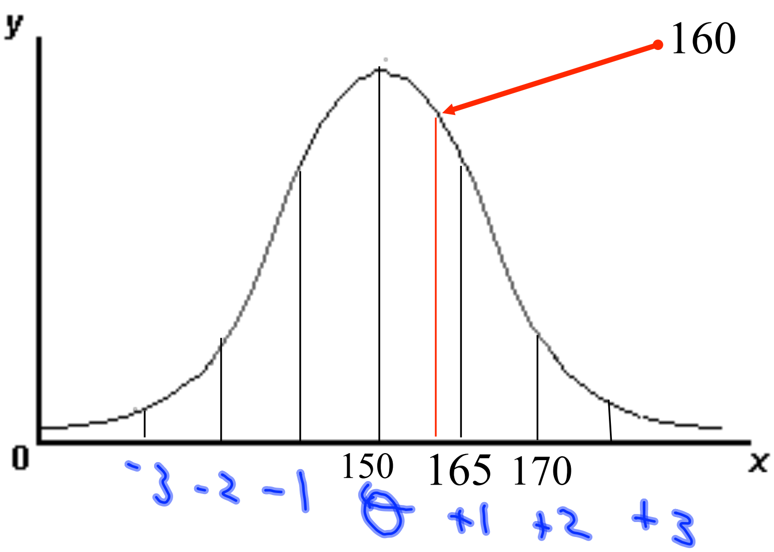
Day 3: The Standard Normal Distribution

What happens if the data in our problems doesn't fall exactly on a standard of deviation?

Eg 1) The heights of students are normally distributed with a mean of 150 cm and a standard deviation of 15 cm. What percentage of the students have a height greater than 160 cm?



If you look at the curve we drew 160 does not fall on one of our standard deviation lines. What do we do?

Normalize the curve by using the z - score formula.

$$z = \frac{x - \mu}{\sigma}$$

$$Z = \frac{160-150}{15}$$

*Round to 2 decimal places

μ (mu) - mean
σ (sigma) - standard deviation
x - measurement or data
z - z-score

So, to finish off example 1... $Z = 0.67$

# of SD to the right or left of our mean.

Eg 2) The annual mean daily temperature for Calgary is 3.5^0 C with a standard deviation of 6.75. The annual mean daily temperature for Regina is 3.1^0 C with a standard deviation of 10.6. If the temperature today was 12^0 C in Regina and 11^0 C in Calgary, which city had the better than average day with respect to temperature?

Calgary 11°C

$Z = \frac{X - \mu}{\sigma}$

$Z = \frac{11 - 3.5}{6.75}$

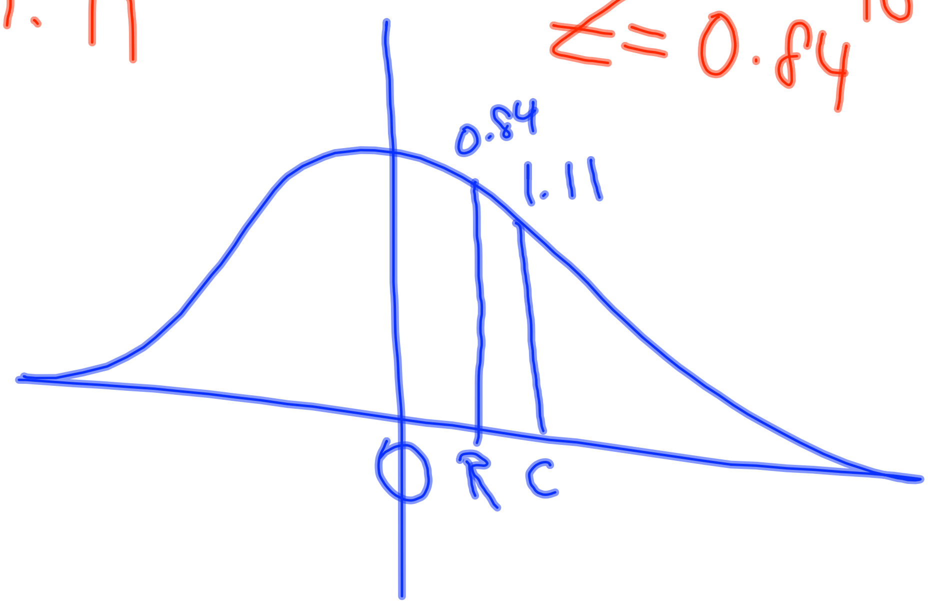
$Z = 1.11$

Regina 12°C

$Z = \frac{X - \mu}{\sigma}$

$Z = \frac{12 - 3.1}{10.6}$

$Z = 0.84$



Eg 3) The results of an exam are found to be normally distributed with a standard deviation of 8.3. Michelle's score of 75 on the exam corresponds to a z-score of 1.35. The mean of the exam is_______________?

$$Z = \frac{x - \mu}{\sigma}$$

$$1.35 = \frac{75 - \mu}{8.3}$$

$$\boxed{\mu = 63.8}$$

Eg 4) A mark of 73 on an exam translates to a z-score of 1.6. If the mean is 64 then the standard deviation to the nearest tenth is _______________?

$$z = \frac{x - \mu}{\sigma}$$

$$1.6 = \frac{73 - 64}{\sigma}$$

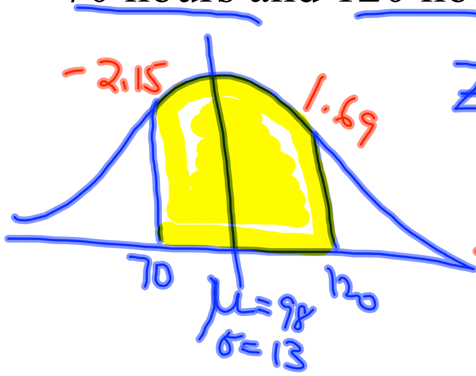
$$\sigma = 5.6$$

When our data does not lie on a standard deviation line we need to be able to determine the percentage of data in the region we are working with.

Eg 5) The Bright light Company tested a new line of light bulbs and found their lifetimes to be normally distributed with a mean life of 98 hours and a standard deviation of 13hours.

a) What percent of the light bulbs lasted between 70 hours and 120 hours.

normalcdf(-2.15, 1.69)



$$Z = \frac{70-98}{13}$$

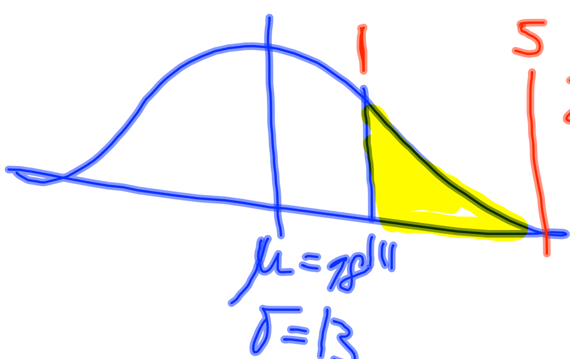
$$Z = -2.15$$

$$Z = \frac{120-98}{13}$$

$$Z = 1.69$$

93.87%

b) What is the probability that a light bulb selected at random will last more than 111 hours?



$$Z = \frac{111-98}{13}$$

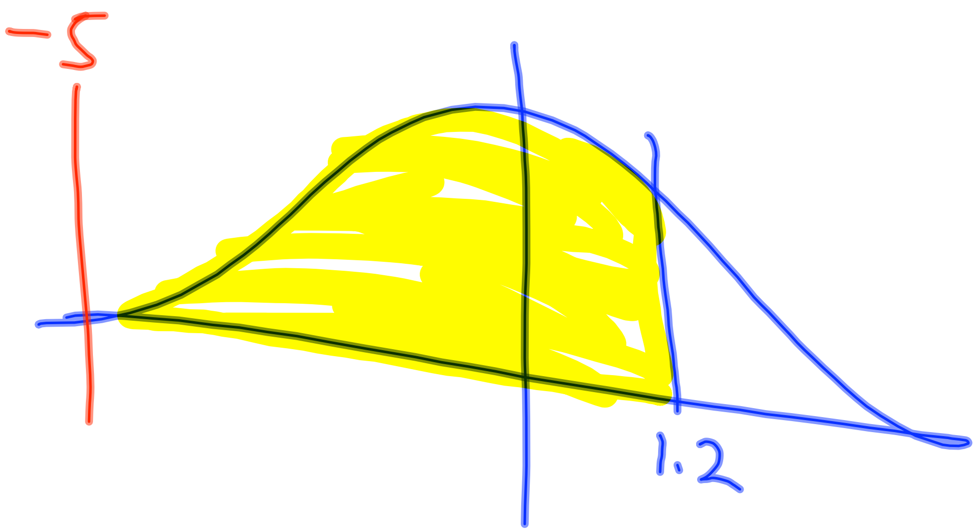
$$Z = 1$$

normalcdf(1, 5)

15.87%

$-1.8 \leq z \leq 4.2$

$z \leq 1.2$



Assignment:
Handout