Geometric Sequences

Let's look at the sequence 1, 3, 9, 27, ...

To get from one term to the next you multiply by 3. This is called having a **common ratio**. A geometric sequence is a sequence in which the terms have a common ratio.

To calculate the common ration you use the following $r = \frac{t_2}{t_1}$

When we work with geometric sequences we use the formula

$$t_n = a(r)^{n-1}$$

a - first term

r - common ratio

n - term number

Find t_5 and t_n for the sequence 4, 8, 16, ...

$t_5 = a(r)^{n-1}$

$t_5 = 4(2)^{5-1}$

$t_5 = 64$

$t_n = 4(2)^{n-1}$

$t_n = 2^2 \cdot (2)^{n-1}$

$t_n = 2^{n+1}$

$t_n = 5(2)^{n-1}$

If -3, 12, -48, ... is a geometric sequence, which term is 3072?

$t_n = a(r)^{n-1}$

$3072 = (-3)(-4)^{n-1}$

$-1024 = (-4)^{n-1}$

$\log(-1024) = (n-1) \cdot \log(-4)$

$\frac{\log(-1024)}{\log(-4)} = n-1$

$5 = n-1$

$n = 6$

The steady growth of a certain bacteria represented by the cell count 4, 16, 64, ...
The number of cels on day 1 is 4, on day 2 is 16 and so on.

a) Write and equation for the growth of these cells.

$t_n = 4^1(4)^{n-1}$

$t_n = 4^n$

b) How many cells are present after 10 days?

$t_{10} = 4^{10}$

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In a geometric sequence, the fourth term is 81 and the sixth term is 729.
Find a, r and t_n.

$t_4 = 81$ $\quad$ $t_6 = 729$

find a, r $\quad$ t_n

$t_n = a(r)^{n-1}$

$81 = a(r)^{4-1}$

$81 = ar^3$

$729 = a(r)^6$

$729 = ar^5$

Divide:

$729 = ar^5$

$81 = ar^3$

$9 = r^2$

$r = \pm 3$

If $r = 3$

$81 = ar^3$

$81 = a(3)^3$

$81 = 27a$

$3 = a$

$t_n = 3^1(3)^{n-1}$

$t_n = 3^n$

If $r = -3$

$81 = a(-3)^3$

$81 = -27a$

$a = -3$

$t_n = (-3)(-3)^{n-1}$

$t_n = (-3)^n$

Terms between the first and last term are called **geometric means**.
In 1, 2, 4, 8, 16, 32 2, 4, 8 and 16 are geometric means.

Find one geometric mean between 1/5 and 125.

$\frac{1}{5}, \_\_, 125$ (r, r)

$\frac{1}{5} \cdot (r) \cdot (r) = 125$

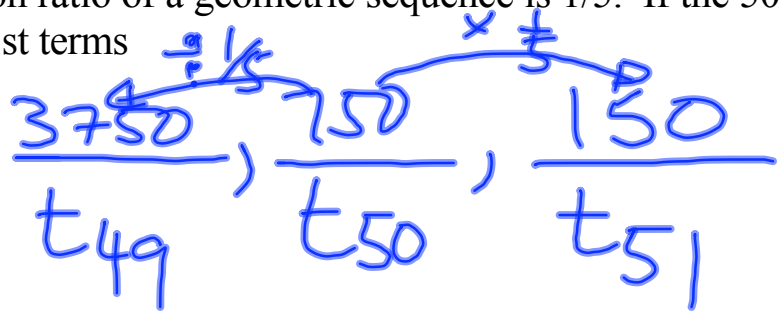
$\frac{1}{5} \cdot r^2 = 125$

$r^2 = 625$

$r = \pm 25$

$\frac{1}{5}, \pm 5, 125$ (×25, −25)

The common ratio of a geometric sequence is 1/5. If the 50th term is 750, find the 49th and 51st terms



Given the following terms in a geometric sequence, find the value of the variable.

(p-4), (2p + 4), (9p + 6)

$r = \frac{t_2}{t_1} = \frac{t_3}{t_2}$

$\frac{2p+4}{p-4} = \frac{9p+6}{2p+4}$

$(2p+4)(2p+4) = (9p+6)(p-4)$

$4p^2 + 16p + 16 = 9p^2 - 30p - 24$

$0 = 5p^2 - 46p - 40$

-50
4
-46

$x -200$

-50, 4

$\frac{-50}{5}$, $\frac{4}{5}$

-10, $\frac{4}{5}$

$(p-10)(5p+4) = 0$

$p - 10 = 0$

$p = 10$

$5p + 4 = 0$

$5p = -4$

$p = \frac{-4}{5}$