Day 4: Basic Trig Identities

Equation: Statement whereby only specific values for x are true.

Eg. 2x + 3 = 5

Identity: Statement whereby all values for x are true.

$$(x+1)^2 = x^2 + 2x + 1$$

Known Trig Identities:

$$\sin\theta = \tfrac{o}{h} = y$$

$$\cos\theta = \tfrac{a}{h} = x$$

$$\tan\theta = \tfrac{o}{a} = \tfrac{y}{x}$$

$$x^2 + y^2 = 1$$

$$\csc\theta = \tfrac{1}{\sin\theta}$$

$$\sec\theta = \tfrac{1}{\cos\theta}$$

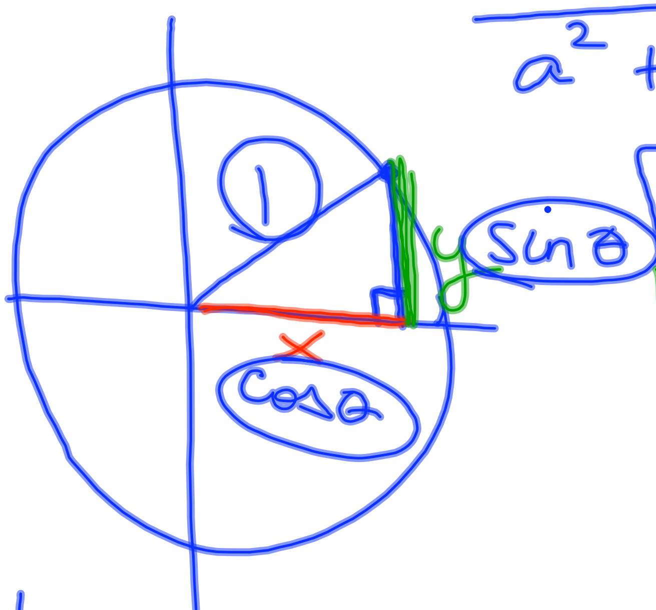
$$\tan\theta = \tfrac{1}{\cot\theta}$$

$$\tan\theta = \tfrac{\sin\theta}{\cos\theta}$$

$$\cot\theta = \tfrac{\cos\theta}{\sin\theta}$$

Pythagorean Identities:

$$\frac{x^2+y^2=1}{a^2+b^2=c^2}$$



$$\boxed{\sin^2\theta+\cos^2\theta=1}$$

$$\sin^2\theta=1-\cos^2\theta$$

$$\cos^2\theta=1-\sin^2\theta$$

$$1+\tan^2x=\sec^2x$$

$$\tan^2x=\sec^2x-1$$

$$1+\cot^2x=\csc^2x$$

$$\cot^2x=\csc^2x-1$$

Consider the identity $\tan x = \boxed{\dfrac{\sin x}{\cos x}}, \cos x \neq 0$

Verify the identity numerically, when $x = \dfrac{\pi}{6}$ and when $x = 4$

$\tan \frac{\pi}{6} = \dfrac{\sin \pi/6}{\cos \pi/6}$

$0.577 = 0.577$ ✓

LS = RS

$\tan 4 = \dfrac{\sin 4}{\cos 4}$

$1.158 = 1.158$

Verify the identity graphically

$\underset{y_1}{\tan x} = \underset{y_2}{\dfrac{\sin x}{\cos x}}$

Prove each of the following (use RS/LS):

$\cos x \tan x = \sin x$

Algebraically Show LS = RS. Transform the LS and/or RS to make the identical.

Hints:

1. NEVER move items from the LS or RS to the opposite side of the eqn.
2. Try to change everything to $\sin\theta$ or $\cos\theta$.

$\cos x \cdot \tan x = \sin x$

$\cancel{\cos x} \cdot \left(\frac{\sin x}{\cancel{\cos x}}\right) = \sin x$

$\sin x = \sin x$ ✓

$$\frac{\cos x \sec x}{\tan x} = \cot x$$

$(\cos x \cdot \sec x) \div (\tan x) = \cot x$

$\underbrace{1 \div \tan x}_{\cot x} = \cot x$

$\cot x = \cot x$ ✓

$$\frac{\sec^2 x}{\sin^2 x} = \frac{1}{\sin^2 x} + \frac{1}{\cos^2 x}$$

$\sec^2 x \div \frac{\sin^2 x}{1} = \frac{1}{\sin^2 x} + \frac{1}{\cos^2 x}$

$\frac{1}{\cos^2 x} \times \frac{1}{\sin^2 x} = \frac{\cos^2 x}{\sin^2 x \cos^2 x} + \frac{\sin^2 x}{\sin^2 x \cos^2 x}$

$\frac{1}{\cos^2 x \cdot \sin^2 x} = \frac{\cos^2 x + \sin^2 x}{\sin^2 x \cos^2 x}$

$= \frac{1}{\sin^2 x \cos^2 x}$ ✓

$$\cos\theta = \frac{1}{\sec\theta} \rightarrow \cos\theta \cdot \sec\theta = 1$$

$$\sin\theta = \frac{1}{\csc\theta} \qquad \sin\theta \cdot \csc\theta = 1$$

$$\tan\theta = \frac{1}{\cot\theta} \qquad \tan\theta \cdot \cot\theta = 1$$

$$\frac{\sin^2 x + \cos^2 x}{\cos x} = \sec x$$

$$\frac{1}{\cos x} = \sec x$$

$$\sec x = \sec x \checkmark$$

$$\sin x + \cos x \underline{\cot x} = \csc x$$

$$\sin x + \cos x \left(\frac{\cos x}{\sin x}\right) = \frac{1}{\sin x}$$

$$* \quad \frac{\sin x}{1} + \frac{\cos^2 x}{\sin x} = \frac{1}{\sin x}$$

$$\frac{\sin^2 x}{\sin x} + \frac{\cos^2 x}{\sin x} = \frac{1}{\sin x}$$

$$\frac{1}{\sin x} = \frac{1}{\sin x}$$

$$\frac{\sin x}{\tan^2 x} = \cot x \cos x$$

$$\frac{\sin x}{1} \div \left(\frac{\sin^2 x}{\cos^2 x}\right) = \left(\frac{\cos x}{\sin x}\right) \cdot \frac{\cos x}{1}$$

$$\frac{\cancel{\sin x}}{1} \times \frac{\cos^2 x}{\sin^2 x} = \frac{\cos^2 x}{\sin x}$$

$$(\cancel{\sin x})(\sin x)$$

$$\frac{\cos^2 x}{\sin x} = \checkmark \frac{\cos^2 x}{\sin x}$$

$$\frac{\sin^2 x}{1 - \cos x} = 1 + \cos x$$

$$\sin^2 x \div (1 - \cos x) = 1 + \cos x$$

$$\underline{(1 - \cos^2 x)} \div (1 - \cos x) = 1 + \cos x$$

$$\frac{(1 + \cos x)\cancel{(1 - \cos x)}}{\cancel{(1 - \cos x)}}$$

$$1 + \cos x \checkmark\!\!= 1 + \cos x$$

Assignment:

Pg. 320 6, 7

Pg. 326 4, 7c, 8c, 10a, 11a, 12-13, 14a, 15a, 16a