Day 5: Sum and Difference Identities

The Sum and Difference Identities (see pg. 330 for derivation):

1. $\sin(\alpha+\beta)=\sin\alpha\cos\beta+\cos\alpha\sin\beta$
2. $\sin(\alpha-\beta)=\sin\alpha\cos\beta-\cos\alpha\sin\beta$
3. $\cos(\alpha+\beta)=\cos\alpha\cos\beta-\sin\alpha\sin\beta$
4. $\cos(\alpha-\beta)=\cos\alpha\cos\beta+\sin\alpha\sin\beta$

1) Expand and simplify:

$$\cos\left(\frac{\pi}{2}+\alpha\right)$$

$\cos(A+B) = \cos A\cos B - \sin A\sin B$

$= \cos\frac{\pi}{2}\cos\alpha - \sin\frac{\pi}{2}\sin\alpha$

$= 0(\cos\alpha) - 1(\sin\alpha)$

$\cos\left(\frac{\pi}{2}+\alpha\right) \rightarrow = -\sin\alpha$

$$\sin\left(\frac{\pi}{2}-\alpha\right)$$

$(\sin\frac{\pi}{2})(\cos\alpha) - (\cos\frac{\pi}{2})(\sin\alpha)$

$\sin(A-B) = \sin A\cos B - \cos A\sin B$

$= 1(\cos\alpha) - 0\sin\alpha$

$= \cos\alpha$

LS = RS

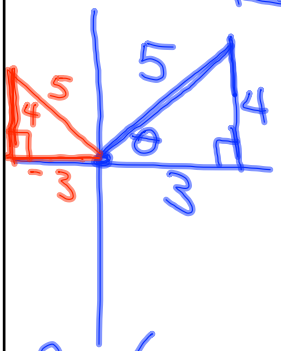
Prove $\cos\left(\frac{3\pi}{2}-\theta\right)=-\sin\theta$

$\cos\frac{3\pi}{2}\cos\theta+\sin\frac{3\pi}{2}\sin\theta=-\sin\theta$

$0(\cos\theta)+(-1)\sin\theta=$

$-\sin\theta=-\sin\theta$ ✓

Given that $\cos\theta=\frac{3}{5}$ and is in Q1, evaluate $\cos\left(\theta+\frac{\pi}{6}\right)$



$a^2+b^2=c^2$

$3^2+b^2=5^2$

$9+b^2=25$

$b^2=16$

$b=4$

$\sin\theta=\frac{o}{h}$

$\cos\left(\theta+\frac{\pi}{6}\right)=\cos\theta\cos\frac{\pi}{6}-\sin\theta\sin\frac{\pi}{6}$

$=\left(\frac{3}{5}\right)\left(\frac{\sqrt{3}}{2}\right)-\left(\frac{4}{5}\right)\left(\frac{1}{2}\right)$

$=\frac{3\sqrt{3}}{10}-\frac{4}{10}$

$=\frac{3\sqrt{3}-4}{10}$

$$\cos\left(\frac{\pi}{2}+\alpha\right) = \boxed{-\sin\theta}$$

a) $\sin\theta$

b) $\cos\theta$

c) $\frac{-1}{\csc\theta}$

d) $\tan\theta$

Evaluate $\sin \frac{5\pi}{12}$

$\frac{5\pi}{12} \times \frac{180}{\pi}$

75°

$\sin(30^\circ + 45^\circ)$

$\sin(A+B) = \sin A \cos B + \cos A \sin B$

$= \sin 30 \cos 45 + \cos 30 \sin 45$

$= \left(\frac{1}{2}\right)\left(\frac{\sqrt{2}}{2}\right) + \left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{2}}{2}\right)$

$= \frac{\sqrt{2}}{4} + \frac{\sqrt{6}}{4}$

$\frac{\sqrt{2}+\sqrt{6}}{4}$

Assignment:

Pg. 333 1, 4, 10c, 11c, 12, 14