

Unit Circle

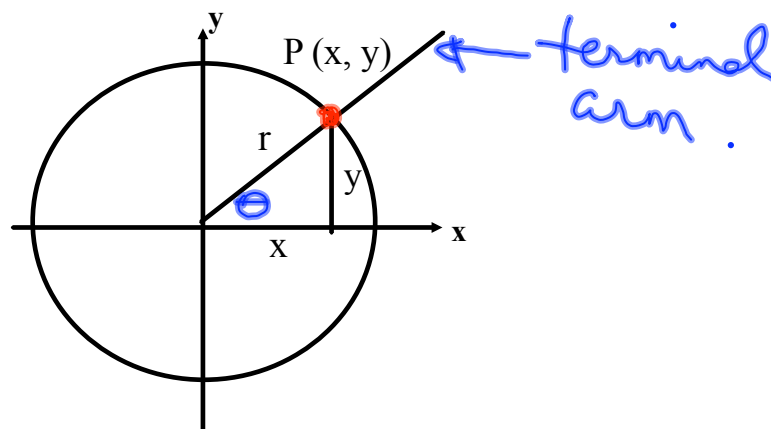
In trigonometry, up to this point, you have studied the primary trigonometric ratios

SOH CAH TOA

$$\sin \theta = \frac{\text{opp}}{\text{hyp}} \quad \cos \theta = \frac{\text{adj}}{\text{hyp}}$$

$$\tan \theta = \frac{\text{opp}}{\text{adj}}$$

We are now going to begin studying the trigonometry of circular functions. To help us do this we need to have extensive knowledge of the **unit circle**. This is a circle drawn in a coordinate plane having a radius of 1 unit.



In a unit circle (where $r = 1$) the primary trigonometric ratios become

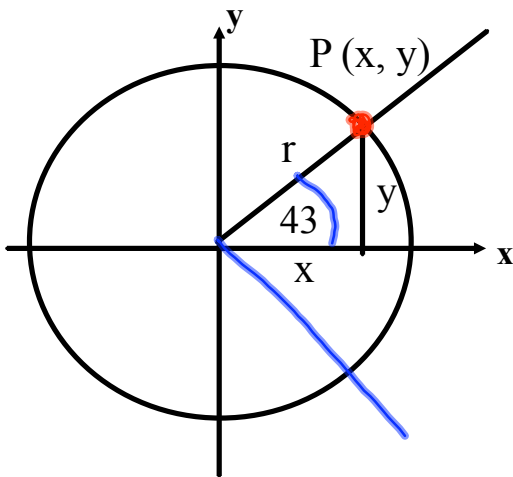
$$\sin \theta = \frac{y}{r} = \frac{y}{1} = y$$

$$\cos \theta = \frac{x}{r} = \frac{x}{1} = x$$

$$\tan \theta = \frac{y}{x}$$

The point $P(x, y)$ represents the point of intersection of the terminal arm of an angle of some measure and the edge of the unit circle.

So, find the point $P(x, y)$ for a 43° angle.



$$x = \cos 43^\circ = 0.7314$$

$$y = \sin 43^\circ = 0.6820$$

$$(0.7314, 0.6820)$$

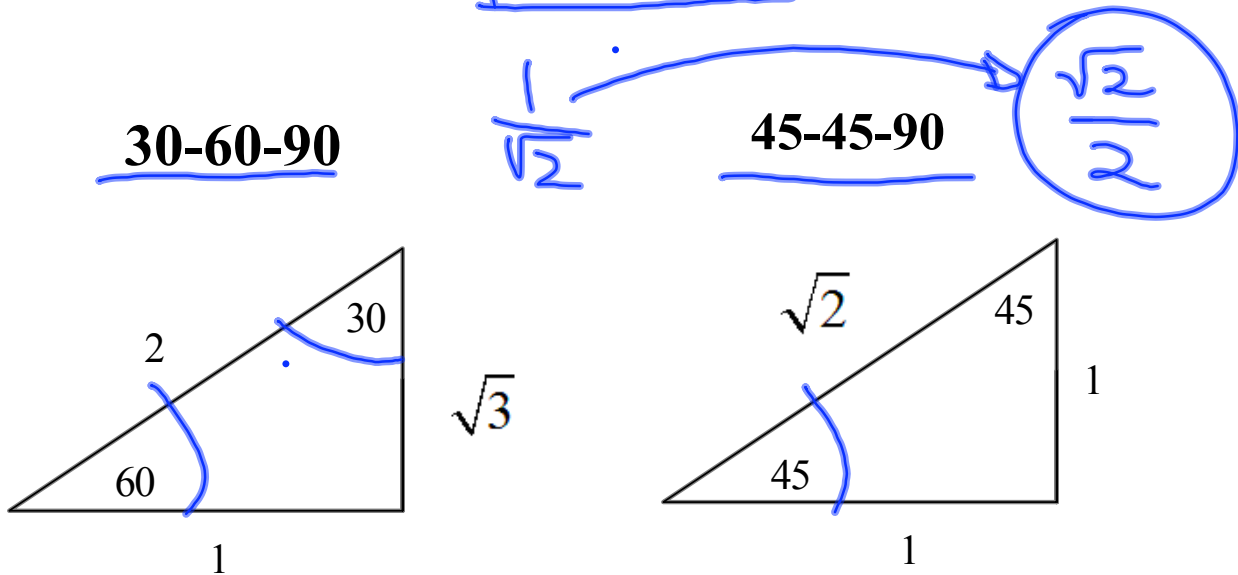
Find the coordinates of the point $P(x, y)$ for an angle of 320°

$$x = \cos 320^\circ$$

$$y = \sin 320^\circ$$

$$(0.7660, -0.6428)$$

We can determine the coordinates of the point $P(x, y)$ for any angle but our answers have been decimals. In mathematics, we often want the result to be an exact value. So, we need to look at **special triangles** to help us.



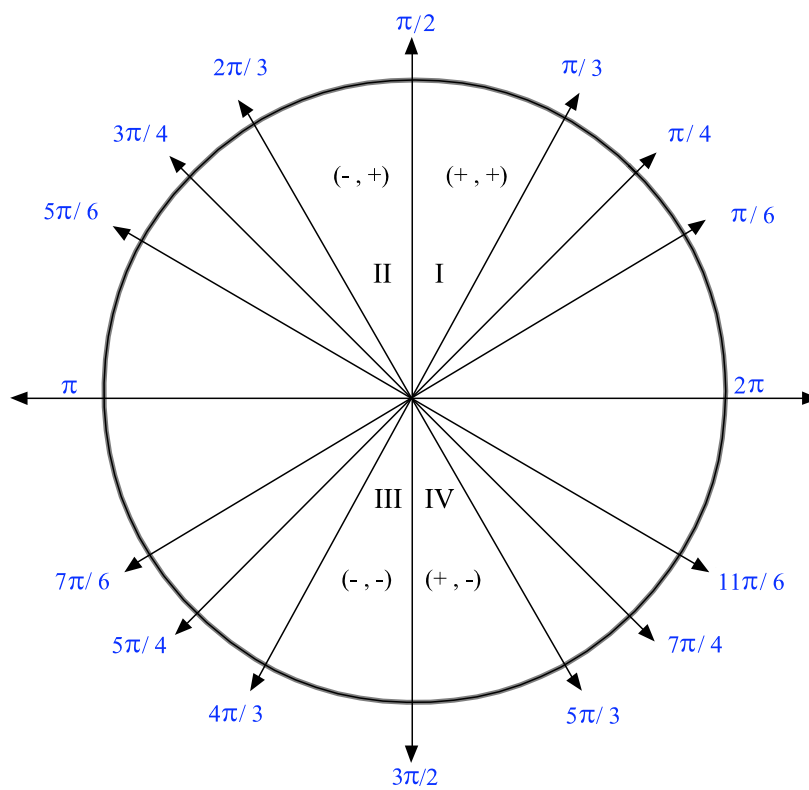
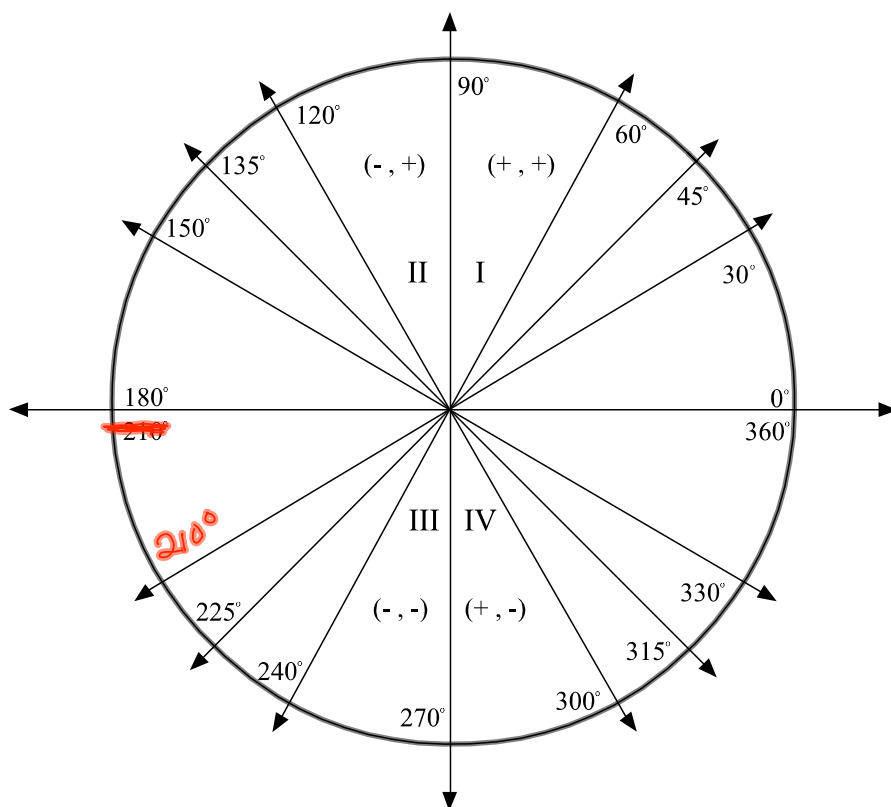
Using these special triangles we can find the **exact value** of any angle in the **unit circle with a measure that is a multiple of 30, 45, 60 or 90 degrees**. For example:

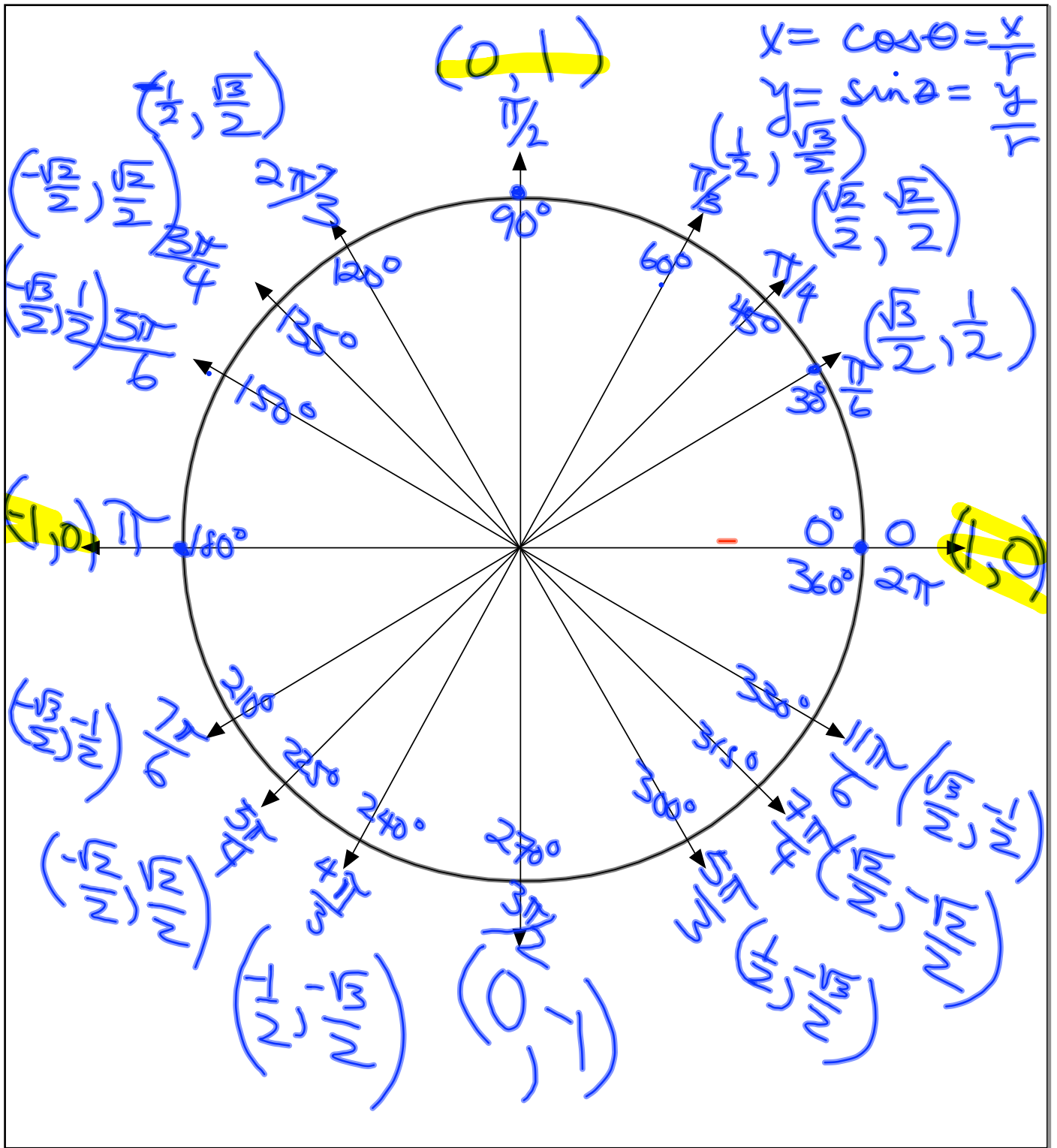
Approximation

$$\sin 60^\circ \doteq \underline{0.866}$$

Exact Value

$$\sin 60^\circ = \frac{\sqrt{3}}{2}$$





Now let's use our unit circle to determine the exact value of the following

$$\sin \frac{2\pi}{3}$$

$$\frac{\sqrt{3}}{2}$$

$$\cos \frac{7\pi}{6}$$

$$-\frac{\sqrt{3}}{2}$$

$$\sin \frac{-5\pi}{4}$$

(135°)

$$\frac{\sqrt{2}}{2}$$

$$\cos \frac{14\pi}{6}$$

$\frac{12\pi}{6} \rightarrow$ one rotation

$$\frac{14\pi}{6} - \frac{12\pi}{6} = \frac{2\pi}{6} = \frac{\pi}{3}$$

$$\frac{1}{2}$$

Fg. 188

3, 4, 6, 8